EFFECT OF IMPURITIES ON LIQUID DISPERSION

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The effect of small amounts of suspended particles with small and large radii on the threshold of wave excitation at the surface of a liquid in a sound field is investigated.

The presence of suspended particles in a water–oil emulsion results in the fact that the waste water from oil fields contains petroleum and oil residues with solid particles suspended in them. In order to utilize the waste water, it is necessary to emulsify the petroleum and oil residues so that the size of their particles does not exceed the dimensions of the pore channels in productive strata. Dispersers with different fragmentation mechanisms are used for this purpose. Thus, a hydrodynamic vibrator [1, 2] whose mechanism for breaking up oil drops is based on the parametric instability of a liquid in a sound field [2, 3] is widely used at present.

We shall consider here the effect of a small amount of suspended particles on the dispersion of a viscous liquid drop in a sound wave field. We assume that the drop radius is much smaller than the wavelength of sound, but also much larger than the capillary radius. The motion of the drop is neglected.

The case under consideration is sufficiently well approximated by the problem of stability of a free surface of an infinite liquid layer containing a small amount of impurities in the field of a plane, vertically incident acoustic wave. This is based on the concept of a cloud of particles as a continuous medium, which has been developed in [4] .

1. We consider a layer of a viscous, incompressible liquid which has the kinematic viscosity coefficient ν, an infinite depth, and a plane free surface. The liquid contains a cloud of spherical nondeformable particles with the same mass m and the radius r. It is assumed that the density of the particle material ρ^0 is much higher than the liquid density ρ. The volumetric percentage of particles is considered to be so low that the interaction between individual particles can be neglected. The Einstein correction of the liquid's viscosity due to the presence of particles can then be neglected, as well as the buoyancy of particles, since these quantities are proportional to the volume concentration of the impurity. The particles are sufficiently large, so that they do not participate in Brownian motion. The analysis is performed in a Cartesian coordinate system whose (x,y) plane coincides with the unperturbed interface, while the z axis points vertically upward. An acoustic wave is incident perpendicularly to the interface, exerting the pressure $p^0 \sin(\omega t + \omega z/c^0)$.

For sufficiently light particles, when their settling and their motion caused by the radiation pressure can be neglected, the equilibrium condition is written thus [5, 6]:

$$\mathbf{v}_0 = 0,\ \mathbf{v}_{01} = 0,\ p_0 = -\rho g z + 2p^0 \sin\left(\omega t + \frac{\omega z}{c}\right),$$
$$p_{02} = p^0\left[\sin\left(\omega t + \frac{\omega z}{c^0}\right) + \sin\left(\omega t - \frac{\omega z}{c^0}\right)\right],\quad \rho_{01} = N_0 m, \tag{1}$$

where $\mathbf{v} = (u, v, w)$, $\mathbf{g} = (0, 0, -g)$ is the acceleration due to gravity, $p_{0(02)}$ is the pressure in the half space $z < 0$ ($z > 0$), and ρ_{01} is the mean density of the particle cloud; the quantities marked by the subscript 1 pertain to the particle cloud.

We shall investigate the stability of this equilibrium by introducing velocity and pressure perturbations in the usual manner. Using $(\alpha/\rho g)^{1/2}$, $(\alpha/\rho g^3)^{1/4}$, $(\alpha g/\rho)^{1/4}$, and $(\alpha g \rho)^{1/2}$ as the units of measurement for length, time, velocity, and pressure, respectively, we obtain for the perturbations the following linearized system of equations [4-9]:

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 36, No. 6, pp. 1012-1017, June, 1979. Original article submitted July 4, 1978.

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla p = \frac{1}{A}\nabla^2 \mathbf{v} + \frac{\mathbf{v}_1 - \mathbf{v}}{\tau}\beta_0, \quad \nabla \mathbf{v} = 0,$$

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{\mathbf{v}_1 - \mathbf{v}}{\tau}, \quad \frac{\partial \beta_1}{\partial t} + \beta_0 \nabla \mathbf{v}_1 = 0, \tag{2}$$

where $A \equiv \nu^{-1}\alpha^{3/4}/(g\rho^3)^{1/4}$; $\beta_{0,1} \equiv \rho_{01,1}/\rho$; $\beta^0 \equiv \rho^0/\rho$; $\tau \equiv (2/9)\beta^0 AR^2$ is the dimensionless time required for the particle velocity relative to the liquid to be reduced by the factor e in comparison with its initial value; $\nabla \equiv (\partial/\partial x, \partial/\partial y, \partial/\partial z)$.

In the case of a small shift of the liquid surface from the equilibrium position ζ, the following expressions must be satisfied at the liquid surface (i.e., for z = 0) [6]:

$$w + \beta_0 w_1 = (1+\beta_0)\frac{\partial \zeta}{\partial t},$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0, \tag{3}$$

$$p = \left[1 - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - q\cos\Omega t\right]\zeta + \frac{2}{A}\frac{\partial w}{\partial z},$$

where $q \equiv 2\Omega P^0/C$.

For $z \to -\infty$,

$$\mathbf{v} \to 0, \ \mathbf{v}_1 \to 0. \tag{4}$$

We have neglected in (3) terms of the order of $q\nabla\zeta$, which are small in comparison with the $\sim q\zeta$ terms, i.e., we assume that the surface oscillations occur with small wave numbers.

By performing Fourier transformation with respect to the x and y variables and the Laplace transformation with respect to time, eliminating the x and y components of the velocity and the pressure, and assuming that the velocity perturbations and the shifts are equal to zero at the initial instant of time and that $\mathbf{v}, \zeta \to 0$; $\partial\mathbf{v}/\partial x$, $\partial\mathbf{v}/\partial y$, $\partial\zeta/\partial x$, $\partial\zeta/\partial y \to 0$ for $|x,y| \to \infty$, we obtain the following instead of (2)-(4):

$$\left(\frac{d^2}{dz^2} - k^2\right)^2 W(s) - As\left(1 + \frac{\beta_0}{1+s\tau}\right)\left(\frac{d^2}{dz^2} - k^2\right)W(s) = 0,$$

$$W_1(s) = W(s)/(1+s\tau), \quad s\beta_1 + \beta_0\nabla \mathbf{V}_1 = 0. \tag{5}$$

For z = 0, we have

$$W(s) + \beta_0 W_1(s) = s(1+\beta_0)Z(s), \quad \frac{d^2W(s)}{dz^2} + k^2 W(s) = 0, \tag{6}$$

$$\frac{1}{k}\left[s\left(1 + \frac{\beta_0}{1+s\tau}\right) - \frac{1}{A}\left(\frac{d^2}{dz^2} - 3k^2\right)\right]\frac{dW(s)}{dz} +$$

$$+ \Omega_0^2 Z(s) - \frac{qk}{2}\left[Z(s+i\Omega + Z(s-i\Omega)\right] = 0, \tag{7}$$

where $\Omega_0^2 = k^3 + k$; $\mathbf{k} = (k_x, k_y)$ is the wave vector and $W(s)$, $\mathbf{V}(s)$, and $Z(s)$ are the Laplace transforms of the quantities $w(t)$, $\mathbf{v}(t)$, and $\zeta(t)$, respectively.

For $z \to -\infty$,

$$W(s) \to 0, \quad W_1(s) \to 0. \tag{8}$$

By solving the system of equations (5) for conditions (6) and (8), we obtain the following expressions for $W(s)$ and $W_1(s)$:

$$W(s) = \frac{(1+\beta_0)Z(s)}{n(\tau)}\left[s + \frac{2k^2}{An(\tau)}\right]\exp(kz) - \frac{2k^2(1+\beta_0)Z(s)}{An^2(\tau)}\exp(\sqrt{k^2 + Asn(\tau)}\,z), \tag{9}$$

$$W_1(s) = W(s)/(1+s\tau), \quad n(\tau) \equiv 1 + \beta_0/(1+s\tau).$$

By substituting the solution (9) in Eq. (7), we obtain the following for $Z(s)$:

$$\left\{\left[s+\frac{2k^2}{An(\tau)}\right]^2-\frac{4k^3\sqrt{k^2+Asn(\tau)}}{A^2n^2(\tau)}+\frac{\Omega_0^2}{1+\beta_0}\right\}Z(s)-\frac{qk}{2(1+\beta_0)}[Z(s+i\Omega)+Z(s-i\Omega)]=0. \quad (10)$$

2. In the case of particles with a small radius R (small values of τ), we expand Eq. (10) in a Taylor series and retain only the linear approximation with respect to τ. After performing inverse Laplace transformation, we obtain the following equation for ζ(t) with an accuracy to R^2:

$$\frac{d^2\zeta}{dt^2}+2\delta(1-\gamma)\frac{d\zeta}{dt}+\left[\frac{\Omega_0^2}{1+\beta_0}+\delta^2-2\gamma-\frac{qk}{1+\beta_0}\cos\Omega t\right]\zeta-$$
$$-\int_0^t\left[e_1\frac{d\zeta(t-\varkappa)}{dt}+e_2\frac{d^2\zeta(t-\varkappa)}{dt^2}\right]\frac{d\varkappa}{\sqrt{\pi\varkappa}}+o\left(\frac{1}{A^{5/2}},\ q\gamma,\ \frac{R^2}{A^2}\right)=0, \quad (11)$$

where $\delta \equiv 2k^2/A(1+\beta_0)$; $\gamma \equiv 4k^2\beta_0\beta^0R^2/9(1+\beta_0)^2$; $e_1 \equiv \sqrt{2}\delta^{3/2}$; $e_2 \equiv 3\gamma\delta^{1/2}/\sqrt{2}$.

Equation (11) constitutes a modification of the Mathieu function [10]. We shall investigate it in the neighborhood of the first instability zone corresponding to the major resonance located near $\Omega_0 = \Omega/2$. Solving (11) by using the averaging method [11], we obtain the following relationship for the boundaries of the instability region:

$$q^2/(1+\beta_0^2)\,k^{-2}[2\delta(1-\gamma)\Omega-e_1\Omega^{1/2}-e_2\Omega^{3/2}]^2-\frac{[2\Omega_0^2/(1+\beta_0)-4\gamma-e_1\Omega^{1/2}-e_2\Omega^{3/2}/2-\Omega^2/2]^2}{[2\delta(1-\gamma)\Omega-e_1\Omega^{1/2}-e_2\Omega^{3/2}]^2}=1, \quad (12)$$

which constitutes the equation of a hyperbola in the plane of the parameters q and Ω_0^2. It characterizes the shift of boundaries of the instability region and the magnitude of this shift as a function of the liquid's viscosity. Analysis of (12) indicates that wave excitation at the surface of a drop resulting in its fragmentation occurs if

$$q\geqslant q_*=\frac{4k\Omega}{A}\left\{1-\frac{k(A\Omega)^{-1/2}}{(1+\beta_0)^{1/2}}-\left[\frac{4k^2\beta_0\beta^0}{9(1+\beta_0)^2}+\frac{k(\Omega A)^{1/2}\beta_0\beta^0}{3(1+\beta_0)^{3/2}}\right]R^2\right\}. \quad (13)$$

Oscillations characterized by the wave number k occur in this case.

The wave number k_* of the most readily excitable waves at the surface is found from the condition $\partial q/\partial k = 0$, whence k_* is determined by using the expression

$$\frac{k_*^3+k_*}{1+\beta_0}-\frac{8k_*^2\beta_0\beta^0R^2}{9(1+\beta_0)^2}=\frac{\Omega^2}{4}+o\left(\frac{1}{A},\ \frac{R^2}{A^{1/2}}\right). \quad (14)$$

It follows from (13) that the presence of suspended particles ($\beta_0 \neq 0$) with a small radius lowers the stability threshold of the liquid drop in comparision with the situation where there are no particles ($\beta_0 = 0$).

3. In the case of large particles (large values of τ), we obtain the following equation for ζ(t) with an accuracy to R^{-2} after expanding Eq. (10) in a Taylor series, retaining only the linear approximation with respect to τ^{-1}, and performing the inverse Laplace transformation:

$$\frac{d^2\zeta}{dt^2}+2\delta_1\frac{d\zeta}{dt}+\left[\frac{\Omega_0^2}{1+\beta_0}+\delta_1^2-\frac{18k^2}{A^2R^2}-\frac{qk}{1+\beta_0}\cos\Omega t\right]\zeta-\int_0^t\left[e_3\frac{d\zeta(t-\varkappa)}{dt}+e_4\zeta(t-\varkappa)\right]\frac{d\varkappa}{\sqrt{\pi\varkappa}}+o\left(\frac{1}{A^3}\right)=0, \quad (15)$$

where $\delta_1 \equiv 2k^2/A$; $e_3 \equiv \sqrt{2}\delta_1^{3/2}$; $e_4 \equiv 2(k/A^{1/2})^5 - 27k^3\beta_0/A^{5/2}R^2\beta^0$.

Solving this equation by means of the averaging method [11] in the vicinity of the first instability region, we obtain the following for the boundaries of the instability region:

$$q^2/(1+\beta_0)^2k^{-2}[2\delta_1\Omega-e_3\Omega^{1/2}+2e_4/\Omega^{1/2}]^2-\frac{[2\Omega_0^2/(1+\beta_0)^{-1}-36k^2/A^2R^2-\Omega^2/2-e_3\Omega^{1/2}-2e_4/\Omega^{1/2}]^2}{[2\delta_1\Omega-e_3\Omega^{1/2}+2e_4/\Omega^{1/2}]^2}=1. \quad (16)$$

Hence we find that wave excitation at the liquid surface, characterized by the wave number k and the frequency Ω/2, occurs if

$$q\geqslant q_*=\frac{4k\Omega(1+\beta_0)}{A}\left[1-\frac{k}{(A\Omega)^{1/2}}+\frac{k^3}{(A\Omega)^{3/2}}-\frac{13.5\,k\beta_0}{(A\Omega)^{3/2}R^2\beta^0}\right]. \quad (17)$$

The wave number k_* of the most readily excitable waves at the liquid surface is determined from the expression

$$\frac{k_*^3+k_*}{1+\beta_0}-\frac{2k_*^3\Omega^{1/2}}{A^{3/2}}+o\left(\frac{1}{A^{5/2}},\ \frac{1}{A^2R^2}\right)=\frac{\Omega^2}{4}. \tag{18}$$

It follows from (17) that the presence of large suspended particles ($\beta_0 \neq 0$) raises the stability threshold of a liquid drop in comparison with the situation where no particles are present, which agrees with the conclusions reached in [9].

Since we solved the problem of disintegration of drops in an acoustic field in the linear approximation, while we did not determine the maximum wave amplitude at the surface, the conditions of wave excitation at the liquid surface (13) and (17) can be interpreted as the lower limit of drop disintegration.

Thus, it is evident that the presence of small particles facilitates liquid dispersion, while large particles inhibit this process. This must be taken into account in calculating dispersers.

NOTATION

α, ν, coefficients of surface tension and kinematic viscosity, respectively; ω, Ω, angular frequency and dimensionless frequency of the acoustic wave, respectively; Ω_0, natural frequency of the wave surface; ρ, ρ_1, and ρ°, densities of the liquid, the particle cloud, and the particle material, respectively; β_1, dimensionless density of the particle cloud; β_0, dimensionless initial density of the particle cloud; β°, dimensionless density of the particle material; τ, characteristic time of interaction between a suspended particle and the medium; δ, δ_1, and γ, dissipative parameters; x, y, z, Cartesian coordinates; t, time; p, pressure; $\mathbf{v} = (u, v, w)$ and $\mathbf{v}_1 = (u_1, v_1, w_1)$, velocity vectors of the liquid and the particle cloud, respectively; k_j, wave number along the j-th axis; p°, P°, dimensional and dimensionless amplitudes of the sonic wave, respectively; c°, c, velocity of sound in air and in the liquid, respectively; C, dimensionless velocity of sound in the liquid; m, particle mass; r, R, dimensional and dimensionless radii of a particle, respectively; N_0, N, mean and actual number of particles per unit volume; A, analog of the Reynolds number; q, small parameter; s, parameter of the Laplace transform; Z(s), W(s), and $V_1(s)$, Laplace transforms for the shift of the surface from the equilibrium position, the z component of the liquid velocity, and the velocity of the particle cloud.

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